1. Show that the product of two Hausdorff spaces is Hausdorff.
2. Show that X is Hausdorff if and only if the diagonal $\Delta := \{(x, x) | x \in X\}$ is closed in $X \times X$.
3. Let (X, d) be a metric space. Show that $d : X \times X \to \mathbb{R}$ is continuous. Show that the topology induced on X by the metric d is the coarsest topology for which $d : X \times X \to \mathbb{R}$ is continuous.
4. A space X is totally disconnected if its only connected subspaces are singletons. Show that X with the discrete topology is a totally disconnected topological space. Give an example of a totally disconnected topological space that is not discrete.
5. Let $f : X \to Y$ be a map. Assume that Y is compact Hausdorff. Then show that f is continuous if and only if the graph of f,
$$\Gamma_f = \{(x, f(x)) | x \in X\},$$
is closed in $X \times Y$.
6. Let $\pi : X \to Y$ be a closed continous surjective map such that $\pi^{-1}(\{y\})$ is compact for each $y \in Y$. Show that if Y is compact then X is compact.