1. Are $(\mathbb{R}, +)$ (real numbers under addition) and $(\mathbb{R}_{>0}, \cdot)$ (strictly positive reals under multiplication) isomorphic as topological groups? Justify your answer.

2. Prove that a subgroup H of a topological group G is open if and only if it contains a non-empty open set.

3. Let G be a topological group. Let G_0 denote the connected component of the identity. Show that G_0 is a closed normal subgroup of G. Show that G/G_0 is a totally disconnected Hausdorff topological group.

4. Let $\phi : G \to H$ be a homomorphism of groups where G and H are topological groups. Show that ϕ is continuous if and only if ϕ is continuous at one point of G.

5. Give an example to show that an injective and surjective map of topological groups need not be an isomorphism of topological groups.

6. Let $A \subset G$ be an abelian subgroup of a topological group G. Show that the closure of A in G denoted by $\overline{A}$ is also an abelian subgroup.

7. Let G be a topological group and let $H \subset G$ be an open subgroup. Show that H is also closed.

8. Let $(\mathbb{R}, +)$ be the topological group where $\mathbb{R}$ is endowed with the standard Euclidean topology. Let $A \subset \mathbb{R}$ be a subgroup. Prove the following:

   (a) If A is discrete then show that $A = a\mathbb{Z}$ for some real number $a \in \mathbb{R}$.

   (b) If A is not a discrete subgroup then show that A is dense in $\mathbb{R}$.

9. Let G be a compact topological group and let $H \subset G$ be a closed subgroup. Show that H is open if and only if H has finite index in G.

10. Prove that $GL_n(\mathbb{C})$ is connected.

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