1. Let $f : X \to Y$ be a continous map. Recall, f is said to be proper if $f^{-1}(C)$ is a compact subset of X for every compact subset $C \subset Y$. Let $f(z) \in \mathbb{C}[z]$ be a polynomial. Let us denote by f the map $z \mapsto f(z)$ from $\mathbb{C} \to \mathbb{C}$. Is this f a proper map? Justify your answer.

   Let $\hat{\mathbb{C}}$ denote the one point compactification of $\mathbb{C}$. Does $f : \mathbb{C} \to \mathbb{C}$ extend to a map $\hat{f} : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$. Justify your answer.

2. Let X be a topological spaces. Show that if X has a countable basis, the collection of subsets $\mathcal{A}$ of X is countably locally finite if and only if it is countable.

3. Let X be a paracompact topological space.

   Claim: Every open covering $\mathcal{A}$ of X has a locally finite subcover.

   Is the above Claim true or false? If true give a proof and if false give a counter-example.

4. Recall a map $p : X \to Y$ is called a perfect map if it is closed, surjective such that $p^{-1}(y)$ is compact for each $y \in Y$. Let $p : X \to Y$ be a perfect map. Show that if Y is paracompact so is X. If X is paracompact Hausdorff show that Y is paracompact.

5. Let G be a locally compact, connected topological group. Show that G is paracompact.

6. Let (X, d) be a metric space. A map $f : X \to X$ is called a contraction if there is a $0 < \alpha < 1$ such that $d(f(x), f(y)) \leq \alpha \cdot d(x, y)$ for all $x, y \in X$. Show that if f is a contraction of a complete metric space then there is a unique $x \in X$ such that $f(x) = x$.

7. Let (Y, d) be a metric space and let $\mathcal{F} \subset C(X, Y)$.

   (a) Show that if $\mathcal{F}$ is finite then $\mathcal{F}$ is equicontinuous.

   (b) Show that if f^n is a sequence of functions that converges uniformly then the collection $\{f^n\}$ is equicontinuous.

   (c) Let $f_n = x^n$ be the function $f_n : [0, 1] \to \mathbb{R}$. Let $\mathcal{F} = \{f_n\}_{n \in \mathbb{N}}$. Is $\mathcal{F} \subset \mathcal{C}(I, \mathbb{R})$ equicontinuous?

8. Let X be a compact n-manifold show that X can be embedded in $\mathbb{R}^m$ for some m.

9. Give an example of an n-manifold that cannot be embedded in $\mathbb{R}^n$.

10. Let $\beta(X)$ denote the Stone-Čech compactification of a topological space X. Show that X is connected if and only if $\beta(X)$ is connected.

28/08/2023