1. Let $f : X \to Y$ be a homotopy equivalence. Show that $f_* : \pi_1(X, x_0) \to \pi_1(Y, f(x_0))$ is an isomorphism.
2. Is every group homomorphism $\pi_1(S^1) \to \pi_1(S^1)$ induced by a map $f : S^1 \to S^1$? Justify your answer.
3. Let $F : X \times I \to X$ be a homotopy from f_0 to f_1, where we follow the notation that $F(x, t) := f_t(x)$. Further assume that $f_0(x) = f_1(x) = x$ for all $x \in X$. Show that for any $x_0 \in X$ the loop $f_t(x_0)$ represents an element in the center of $\pi_1(X, x_0)$.
4. Let $X := \mathbb{R}^n - \{x_1, \dots, x_m\}$ where $x_i \in \mathbb{R}^n$. Compute $\pi_1(X)$ when $n \geq 2$.
5. Let $\tilde{X} \to X$ be a covering space. Let $A \subset X$ and let $\tilde{A} = p^{-1}(A)$. Show that $p|_{\tilde{A}} : \tilde{A} \to A$ is a covering space. Let us further assume that $p^{-1}(x)$ is a finite set for all $x \in X$. Show that $\tilde{X}$ is compact Hausdorff if and only if X is compact Hausdorff.
6. Let X be a path connected, locally path connected topological space. Suppose that $\pi_1(X)$ is a finite group. Show that every map from $X \to S^1$ is nullhomotopic.
7. Classify all connected two sheeted coverings of $S^1 \vee S^1$ upto isomorphism.
8. Let G be a finitely generated abelian group. Construct a topological space X such that $\pi_1(X) \cong G$.

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