1. Show that a complement of a finite set of points in $\mathbb{R}^n$ is simply connected if $n \geq 3$.
2. Let $X \subset \mathbb{R}^3$ be the union of n lines through the origin. Compute $\pi_1(\mathbb{R}^3 - X)$.
3. Compute π_1 of the Mobius strip.
4. Find all connected covering spaces of $\mathbb{R}P^2 \vee \mathbb{R}P^2$.
5. Given a group G and a normal subgroup N show that there exists a normal covering space $\widetilde{X} \to X$ with $\pi_1(X) \cong G$ and $\pi_1(\widetilde{X}) \cong N$ and deck transformation group isomorphic to G/N.
6. Show that $\widetilde{H}_n(X) \cong \widetilde{H}_{n+1}(SX)$ for all n where SX denotes the suspension of X.
7. Let $f : S^n \to S^n$ be a map of degree zero. Show that there exists $x, y \in S^n$ with $f(x) = x$ and $f(y) = -y$.
8. Construct a surjective map $S^n \to S^n$ of degree zero for each $n \geq 1$.
9. Let $f(z)$ be a polynomial with complex coefficients, which can be seen as a map from $\mathbb{C} \to \mathbb{C}$. Let $\hat{f} : S^2 \to S^2$ be the extension of f to the one-point compactification. Show that the degree of $\hat{f}$ equals the degree of f as a polynomial.
10. A map $f : S^n \to S^n$ satisfying $f(x) = f(-x)$ for all x is called an even map. Show that an even map $S^n \to S^n$ must have an even degree. When n is even shown the degree must be zero. When n is odd show that there exists an even map of any given even degree.
11. For a finite CW complex show that $\chi(X \times Y) = \chi(X) \times \chi(Y)$.
12. For X a finite CW complex and $p : \widetilde{X} \to X$ a n-sheeted covering map, show that $\chi(\widetilde{X}) = n\chi(X)$.
13. Let M_i denote a closed orientable surface of genus i. Show that if M_g is a covering of M_h then $g = n(h-1) + 1$ where n is the number of sheets of the covering. If $g = n(h-1) + 1$ is there always an n sheeted covering $M_g \to M_h$?
14. Let $T_n(X, A)$ denote the torsion subgroup of $H_n(X, A)$. Does the functor $(X, A) \mapsto T_n(X, A)$ define a homology? Justify your answer. What about $MT_n(X, A) := H_n(X, A)/T_n(X, A)$.
15. Show that there are only countably many homotopy types of a finite CW complex.

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